

Discrete Fourier Transform

Background

In digital signal processing, the frequency components of a sampled signal can be computed using the Discrete Fourier Transform (DFT). A continuous (analog) signal $x(t)$ is sampled at a frequency of f_s Hz, to obtain the sampled signal $x(n)$, where $n = 0, \dots, N-1$ and N is the number of samples. The sequence $x(n)$ can then be transformed into $X(k)$, where $k = 0, \dots, N-1$, by DFT according the formula:

$$\begin{aligned} X(k) &= \sum_{n=0}^{N-1} x(n) e^{-\frac{2\pi j}{N} kn} \\ &= \sum_{n=0}^{N-1} x(n) \left[\cos\left(\frac{2\pi kn}{N}\right) - j \sin\left(\frac{2\pi kn}{N}\right) \right] \end{aligned}$$

The value of $X(k)$ corresponds to the frequency component of $x(n)$ at frequency bin k , where $\frac{k \cdot f_s}{2N} \leq f < \frac{(k+1) \cdot f_s}{2N}$. It is important to note that $X(k)$ can only describe the frequency components up to $f < \frac{f_s}{2}$ according to the Nyquist-Shannon sampling theorem.

Let

$$c(k) = \sum_{n=0}^{N-1} x(n) \cos\left(\frac{2\pi kn}{N}\right) \text{ and } s(k) = \sum_{n=0}^{N-1} x(n) \sin\left(\frac{2\pi kn}{N}\right)$$

The magnitude of $X(k)$ is given as:

$$|X(k)| = \sqrt{c^2(k) + s^2(k)}$$

The phase $X(k)$ is given as:

$$\angle X(k) = \tan^{-1} \frac{c(k)}{s(k)}$$

Problem Statement

Given a text file representing a sampled signal, you are to write a program in C that computes the DFT of the signal and report the magnitude and phase at k . You can make use of C math library functions such as sine, cosine, square root, and arc tangent.

Requirements:

1) Your program must be well documented (See the example).

2) Your program must be modular. Your program must include and use the following functions with the exact same arguments:

a) Function DFT—computes the real and imaginary parts, c and s, respectively, of the sequence x at k. Data set x contains N samples.

```
void DFT(double* c, double* s, double *x, int N, int k);
```

Note: this function uses pointers; no array operations are allowed here.

b) Function ComputeMagnitude—computes the magnitude of a complex number.

```
double ComputeMagnitude(double real, double imag);
```

c) Function ComputePhase—computes the phase in radians of a complex number.

```
double ComputePhase(double real, double imag));
```

Submission

1. You may get 3% bonus if you submit your assignment 24 hours prior to the due date/time.
2. Demonstrate your program to the TA in the lab on or before the due date
3. Remove all .exe file from your project folder.
4. Zip (No RAR, please) your project folder and rename it to **ECE4220P1_YourLastname_Firstname.zip**.
5. Submit the zipped file to the TA using Digital Drop Box in Blackboard.
6. For late submission, five percent per day will be deducted. After one week, no late submission will be accepted.

Program Output Example

```
ECE4220 Assignment #1 Digital Fourier Transform
```

```
=====
```

```
Please enter the name of the data file for signal x(n): data1.txt
```

```
Reading... 1024 data points read.
```

```
To compute the DFT X(k), please enter k: 0
```

```
The magnitude is 2.1 units.
```

```
The phase is 1.5 radians.
```

```
Do you want to compute another X(k)? (Y/N): y
```

```
To compute the DFT X(k), please enter k: 5
```

```
The magnitude is 0.2 units.
```

```
The phase is 0.9 radians.
```

```
Do you want to compute another X(k)? (Y/N): n
```

```
Good bye!
```

Note: Bold fonts indicate user inputs.