

Matrix Inversion

Note Title

2/13/2005

$$A^{-1} = \frac{C_{ij}^T}{|A|}$$

where cofactor, $C_{ij} = (-1)^{i+j} M_{ij}$

M_{ij} is the minor of matrix A (eliminate row i + col. j)

$$\text{determinant, } |A| = \sum_{i=1}^k a_{ij} C_{ij}$$

GENERAL 3x3 Matrix

$$A = \begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix}$$

$$|A| = aei - ahf - dbi + dhc + gbf - gec$$

Finding the Cofactor:

$A =$

$i=1$	a	d	g
2	b	e	h
3	c	f	i

$\text{cofactor} =$

$$\begin{bmatrix} (-1)^{1+1} \det \begin{pmatrix} e & h \\ f & i \end{pmatrix} & \dots & (-1)^{1+3} \det \begin{pmatrix} b & e \\ c & f \end{pmatrix} \\ \vdots & & \vdots \\ (-1)^{3+1} \det \begin{pmatrix} d & g \\ e & h \end{pmatrix} & \dots & (-1)^{3+3} \det \begin{pmatrix} a & d \\ b & e \end{pmatrix} \end{bmatrix}$$

EXAMPLE

$$A = \begin{bmatrix} 1 & 0 & l \\ 1 & 0 & -l \\ 0 & 1 & 0 \end{bmatrix}; \quad |A| = 0 + l - 0 + 0 + l - 0 \\ = 2l$$

cofactor, $C_{ij} = \begin{bmatrix} +l & -0 & +1 \\ +l & +0 & -1 \\ +0 & +2l & +0 \end{bmatrix}$

$$A^{-1} = \frac{C_{ij}^T}{|A|} = \frac{\begin{bmatrix} l & l & 0 \\ 0 & 0 & 2l \\ 1 & -1 & 0 \end{bmatrix}}{2l} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \\ \frac{1}{2l} & \frac{-1}{2l} & 0 \end{bmatrix}$$